$\hat{\mathbf{x}}$ direction, as we can tell from the fact that $\mathbf{E} \times \mathbf{B}$ points in that direction. Using Eqs. 30 and the identity $\gamma^2(1 - \beta^2) = 1$, we find that

$$E'_y = E_0 \sqrt{\frac{1-\beta}{1+\beta}} \qquad B'_z = E_0 \sqrt{\frac{1-\beta}{1+\beta}} \tag{33}$$

As observed in F' the amplitude of the wave is reduced. The wave velocity, of course, is c in F', as it is in F. The electromagnetic wave has no rest frame. In the limit $\beta = 1$, the amplitudes E'_y and B'_z observed in F' are reduced to zero. The wave has vanished!

PROBLEMS

9.1 If the electric field in free space is $\mathbf{E} = E_0(\hat{\mathbf{x}} + \hat{\mathbf{y}}) \sin (2\pi/\lambda)(z + ct)$, with $E_0 = 2$ statvolts/cm, the magnetic field, not including any static magnetic field, must be what?

9.2 The power density in sunlight, at the earth, is roughly 1 kilowatt/meter^2. How large is the rms magnetic field strength?

Ans. 0.02 gauss or 2×10^{-6} tesla.

9.3 A free proton was at rest at the origin before the wave described by Eq. 22 came past. Where would you expect to find the proton at time $t = 1$ microsecond? The pulse amplitude is in statvolts/cm. Proton mass $= 1.6 \times 10^{-24}$ gm. *Hint:* Since the duration of the pulse is only a few nanoseconds, you can neglect the displacement of the proton during the passage of the pulse. Also, if the velocity of the proton is not too large, you may ignore the effect of the magnetic field on its motion. The first thing to calculate is the momentum acquired by the proton during the pulse.

9.4 Suppose that in the preceding problem the effect of the magnetic field was not entirely negligible. How would it change the direction of the proton's final velocity?

9.5 Here is a particular electromagnetic field in free space:

$$E_x = 0 \qquad E_y = E_0 \sin(kx + \omega t) \qquad E_z = 0$$

$$B_x = 0 \qquad B_y = 0 \qquad B_z = -E_0 \sin(kx + \omega t)$$

(*a*) Show that this field can satisfy Maxwell's equations if ω and k are related in a certain way.

(*b*) Suppose $\omega = 10^{10}$ sec^{-1} and $E_0 = 0.05$ statvolt/cm. What is the wavelength in cm? What is the energy density in ergs/cm^3, averaged over a large region? From this calculate the power density, the energy flow in ergs/cm^2-sec.

9.6 Start with the source free, or "empty space" Maxwell's equations in SI units, obtained by dropping the terms with ρ and **J** from Eq. 15′. Consider a wave described by Eqs. 17 and 18, but now with E_0 in volts/meter and B_0 in teslas. What conditions must E_0, B_0, and v meet to satisfy Maxwell's equations?

9.7 Write out formulas for **E** and **B** that specify a plane electromagnetic sinusoidal wave with the following characteristics. The wave is traveling in the direction $-\hat{\mathbf{x}}$; its frequency is 100 megahertz (MHz, 10^8 cycles per sec); the electric field is perpendicular to the $\hat{\mathbf{z}}$ direction.

9.8 Show that the electromagnetic field described by

$$\mathbf{E} = E_0\hat{\mathbf{z}} \cos kx \cos ky \cos \omega t$$

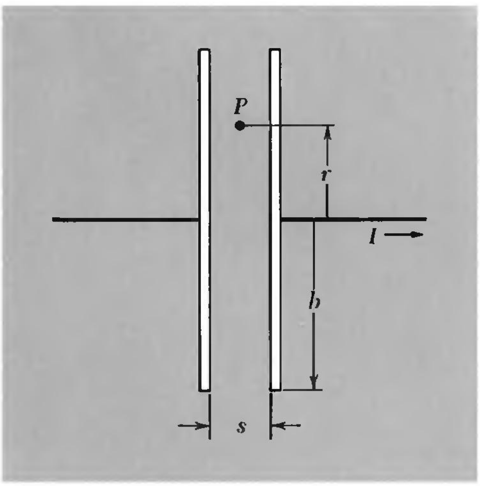
$$\mathbf{B} = B_0(\hat{\mathbf{x}} \cos kx \sin ky - \hat{\mathbf{y}} \sin kx \cos ky) \sin \omega t$$

will satisfy Eqs. 16 if $E_0 = \sqrt{2}B_0$ and $\omega = \sqrt{2}ck$. This field can exist inside a square metal box, of dimension π/k in the x and y directions and arbitrary height. What does the magnetic field look like?

9.9 Of all the electromagnetic energy in the universe, by far the largest amount is in the form of waves with wavelengths in the millimeter range. This is the cosmic microwave background radiation discovered by Penzias and Wilson in 1965. It apparently fills all space, including the vast space between galaxies, with an energy density of 4×10^{-13} erg/cm^3. Calculate the rms electric field strength in this radiation, in statvolts/cm, and convert it to volts/meter. Roughly how far away from a 1-kilowatt radio transmitter would you find a comparable electromagnetic wave intensity?

Ans. 0.06 volt/meter; 3 km.

PROBLEM 9.10



9.10 The magnetic field inside the discharging capacitor shown in Fig. 9.1 can in principle be calculated by summing the contributions from all elements of conduction current, as indicated in Fig. 9.5. That might be a long job. If we can assume symmetry about this axis, it is very much easier to find the field **B** at a point by using the integral law

$$\int_C \mathbf{B} \cdot d\mathbf{s} = \frac{1}{c} \int_S \left(\frac{\partial \mathbf{E}}{\partial t} + 4\pi \mathbf{J} \right) \cdot d\mathbf{a}$$

applied to a circular path through the point. We need only know the total current enclosed by this path. Use this to find the field at P, which is midway between the capacitor plates and a distance r from the axis of symmetry. (Compare this with the calculation of the induced electric field **E**, in the example of Fig. 7.16.)

$$\textit{Ans.}\ 2\pi r B = \frac{4\pi I}{c}\frac{r^2}{b^2},\ B = \frac{2Ir}{cb^2}.$$

9.11 From a satellite in stationary orbit a signal is beamed earthward with a power of 10 kilowatts and a beam width covering a region roughly circular and 1000 km in diameter. What is the electric field strength at the receivers, in millivolts/meter?

9.12 A sinusoidal wave is reflected at the surface of a medium whose properties are such that half the incident energy is absorbed. Consider the field that results from the superposition of the incident and the reflected wave. An observer stationed somewhere in this field finds the local electric field oscillating with a certain amplitude E. What is the ratio of the largest such amplitude noted by an observer to the smallest amplitude noted by any observer? (This is called the *voltage standing wave ratio,* in laboratory jargon, VSWR.)

9.13 Starting from the field transformation given by Eq. 60 of Chapter 6, show that the scalar quantity $E^2 - B^2$ is invariant under the transformation. In other words, show that $E'^2 - B'^2 = E^2 - B^2$. You can do this using only vector algebra, without writing out x, y, z components of anything. (The resolution into parallel and perpendicular vectors is convenient for this, since $\mathbf{E}_\perp \cdot \mathbf{E}_\parallel = 0$, $\mathbf{B}_\parallel \times \mathbf{E}_\parallel = 0$, etc.)